Controlling light and matter using cooperative radiation

Part I: Dicke states, cooperative effects, entanglement

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Herrsching, March 6, 2019
Roy Glauber, 1925 - 2018
Goals of the lectures

- Use 2D array...
• ... to reflect light ...
• ... emit in controlled way ...
Increase (impurity) cross section?
Increase (impurity) cross section?
... edge states with photons?
• 2D materials like graphene or TMDs
• ... to switch
  (single photons)
• ... to switch (single photons)
Quantum mirror: Refraction superposition
Why “cooperative effects”??
Cooperative radiation: superradiance

For want of a better term, a gas which is radiating strongly because of coherence will be called “superradiant.”

What is an atom?

levels

_____  

_____  

_____  

_____
What is an atom?

(levels)

(sub)levels

____

____

____

____
How does it couple to light?

photon energy: $h\nu$

atomic transition energy: $E_{up} - E_{down}$
How does it couple to light?

Photon energy: $h\nu$

Atomic transition energy: $E_{up} - E_{down}$
Cooperative effects in radiation

single atom
Cooperative effects in radiation

two far-away atoms
Cooperative effects in radiation

two close atoms

![Graph showing cooperative effects in radiation with time on the x-axis and intensity on the y-axis. The graph compares the intensity of radiation from two close atoms over time, with a solid line representing one set of observations and a dashed line representing another set. The diagram illustrates the decay of radiation intensity with time.]
Cooperative effects in radiation

- Superradiance $\propto N^2$
due to constructive interference
- Build-up of “collective dipoles”

many close atoms

![Diagram showing time and intensity with a peak due to superradiance and the build-up of collective dipoles.](image-url)
Cooperative Effects

• Two important aspects:
  – collective effects (many particles)
Cooperative Effects

- Two important aspects:
  - collective effects (many particles)
    - e.g., much higher chance for photons to interact with many atoms than one
  - “exchange” due to dipole-dipole interaction
    - excitations are exchanged
- “Cooperative” is more than just “collective”!
Collective effects: Example of quantized field

- Interaction Hamiltonian with classical light (Rabi frequency $\Omega$)

$$\frac{H}{\hbar} = \Omega (|e\rangle\langle g| + |g\rangle\langle e|)$$

- Interaction Hamiltonian with quantized light (coupling element $g$, annihilation operator $a$, number of atoms $N$)

$$\frac{H}{\hbar} = g \sqrt{N} (|e\rangle\langle g| a + a^+ |g\rangle\langle e|)$$
Cooperative effects

• simplest form of “exchange interaction”
Cooperative effects

- Traditional example: superradiance
Cooperative effects in radiation

many close atoms

- Superradiance $\propto N^2$ due to constructive interference
- Build-up of “collective dipoles”
What is **super** in superradiance?

**man** (Clark)

**many men** (Clarks)
What is **super** in superradiance?

man (Clark)

many men (Clarks)
What is **super** in superradiance?

- man (Clark)
- many men (Clarks)
Examples
Superradiant laser

# Atoms $\gg$ # Photons

much more coherent lasing

Super-radiance in BECs

momentum conservation + interference of matter waves

Inouye, Chikkatur, Stamper-Kurn, Stenger, Pritchard, Ketterle
Science 23 (1999)
rotation energy amplifies waves if $\Omega_{\text{rot}} > \omega_{\text{wave}}$

Whirling around: waves scatter from a vortex

Black hole superradiance

spinning black hole…. 

… amplifies gravitational waves  
(basically same as for water waves)

Pani, Brito, Cardoso, Class. Quantum Grav. 32 134001 (2015)
Subradiance

pulse
no pulse

These lectures

• Cooperative effects in complex systems

• New application: atomically thin mirrors
These lectures

- Cooperative effects in complex systems
  - Collective (Lamb) level shifts
  - Subradiance
  - Entanglement

- New application: atomically thin mirrors
These lectures

• Cooperative effects in complex systems

• New application: atomically thin mirrors
  ‣ Cooperative resonances
  ‣ Applications:
These lectures

• Cooperative effects in complex systems

• New application: atomically thin mirrors
  ‣ Cooperative resonances
  ‣ Applications:
    • topology with photons
    • nonlinear quantum optics
    • Quantum metasurfaces
Superradiance: recent experiment

Grimes, Coy, Barnum, Zhou, Yelin, Field, PRA 95, 043818 (2017)
Superradiance: recent experiment

Grimes, Coy, Barnum, Zhou, Yelin, Field, PRA 95, 043818 (2017)
Superradiance in Ba Rydbergs

Collective shift

Typical superradiance time profile

Shift from red...
Superradiance in Ba Rydbergs

Collective shift

typical superradiance time profile
Cooperative radiation: two atoms

atoms distinguishable

\[ |ee\rangle \]
\[ |eg\rangle \quad |ge\rangle \]
\[ |gg\rangle \]
Cooperative radiation: two atoms

atoms indistinguishable

\[ \frac{1}{\sqrt{2}} (|eg\rangle + |ge\rangle) \equiv |1, 0\rangle \]

\[ |ee\rangle \equiv |1, 1\rangle \]

\[ \frac{1}{\sqrt{2}} (|eg\rangle - |ge\rangle) \equiv |0, 0\rangle \]

\[ |gg\rangle \equiv |1, -1\rangle \]
Cooperative radiation: two atoms

Atoms indistinguishable

\[
\frac{1}{\sqrt{2}} (|eg\rangle + |ge\rangle) \equiv |1, 0\rangle
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|gg\rangle \equiv |1, -1\rangle

destructive interference
Cooperative radiation: two atoms

atoms indistinguishable

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dipole-dipole exchange interaction
Cooperative radiation: two atoms

Atoms indistinguishable

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dipole-dipole exchange interaction
Cooperative radiation: two atoms

atoms indistinguishable

\[ |e e\rangle \equiv |1, 1\rangle \]

\[ \frac{1}{\sqrt{2}} (|e g\rangle + |g e\rangle) \equiv |1, 0\rangle \]

exchange interaction:

- usually dipole-dipole mediated
- creates shift and broadening (Kramers-Kronig)
What is “superradiance”?

1. Everything that involves Dicke states
   - (e.g., collective VN effects,
   - bad-cavity limit,
   - ...)


Dicke states

Fully symmetric state of $n$ excitations in $N$ particles, for example

$$|2\rangle_4 = \frac{1}{\sqrt{6}} (|1100\rangle + |1010\rangle + |1001\rangle + |0110\rangle + |0101\rangle + |0011\rangle)$$

$N$-particle Dicke states decay with up to $N^2$ speedup
Dicke states

• Question: When do we have a system that consists only of Dicke states?

• Answer 1: When there exists no mechanism to distinguish atoms. Example:

• Problem: interactions (dip-dip) drive system out of purely symmetric state! How to deal?

• (Answer 2: When the exchange interaction is infinitely high. In this case, other states cannot be reached. Example: “many-body protected manifolds”)
Fig. 1. Energy level diagram of an \( n \)-molecule gas, each molecule having 2 nondegenerate energy levels. Spontaneous radiation rates are indicated. \( E_m = mE \).
Describing superradiance

Use angular momentum form:

$| J, m \rangle$ denotes system with $m$ excitations, $m = -J \ldots J$.

Fig. 1. Energy level diagram of an $n$-molecule gas, each molecule having 2 nondegenerate energy levels. Spontaneous radiation rates are indicated. $E_m = mE$. 
Angular momentum states

• Form of 3-atom Dicke state?
• What are the other states?
• 4 atoms?
• 40?
Population of symmetric states

- What possible path could a system starting in $|\underbrace{11111}_{\text{\ldots}}\rangle$ take,
- with only decay?
- when there is dipole-dipole interaction?
State connections

\[ J = \frac{N}{2} \]

\[ J = \frac{N}{2} - 1 \]

\[ J = 0 \]

Spontaneous superradiant decay
State connections

\[ J = \frac{N}{2} \quad J = \frac{N}{2} - 1 \quad J = 0 \]

Spontaneous superradiant decay
dipole-dipole interaction
Form of dipole-dipole interaction

\[ V_{\text{dip-dip}} \propto \sum_{i \neq j} \frac{e^{i \theta_{ij}}}{x_{ij}^3} \left[ (1 - \cos^2 \theta_{ij}) x_{ij}^2 + (1 - 3 \cos^2 \theta_{ij}) (ix_{ij} - 1) \right] \]

\[ H_{\text{dip-dip}} = \sum_{i \neq j} V_{ij} \sigma_i^+ \sigma_j^- \]
“Exchange” term

• How would this show up in a master equation?
Form of dipole-dipole interaction

\[ V_{\text{dip-dip}} \propto \sum_{i \neq j} \frac{e^{i\theta_{ij}}}{x_{ij}^3} \left[ (1 - \cos^2 \theta_{ij}) x_{ij}^2 + (1 - 3 \cos^2 \theta_{ij}) (ix_{ij} - 1) \right] \]

- dipole-dipole interaction:
  - distance dependence
  - angle dependence
  - real + imaginary part

Virtual photon exchange

Real photon exchange

Flip-flop "exchange"

Shift/dephasing
Atom-atom correlations in superradiance: Classic example

- Superradiance

Gross, Haroche, Phys. Rep. 93, 301 (‘82)
Same parameters as before: $C=10$ with exchange
generates a peak in the intensity.


The graph shows the intensity per atom as a function of time normalized by the decay rate ($t/\gamma^{-1}$). Two curves are plotted:

- **Red curve (with exchange):** The intensity decays sharply and quickly.
- **Blue curve (no exchange):** The intensity decays more slowly, with a pronounced peak, indicating "amplified spontaneous emission".
What is “superradiance”?

1. Everything that involves Dicke states
   – (e.g., collective VN effects,
   – bad-cavity limit,
   – ...)

2. Only systems involving cooperative (and nonlinear) effects
   – i.e., effect of exchange interaction
   – more than single excitation
What is “superradiance”?

1. Everything that involves Dicke states
   - (e.g., collective V N effects,
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   - ...)

2. Only systems involving cooperative (and nonlinear) effects
   - i.e., effect of exchange interactions
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only for purists
Full dynamics (all degrees of freedom of atoms, fields)

two probe atoms
+
surrounding atoms

Two atoms + field

effective two-atom description

Dynamics of atoms in dense media - Schwinger-Keldysh & Dyson Eq.

Full dynamics (all degrees of freedom of atoms, fields)

$$H = H_{\text{atoms}} + H_{\text{field}} - \sum p_i E_i$$

two probe atoms
+ surrounding atoms

Two atoms + field

$$V_{\text{probe}} = \sum_{i=1,2} p_i E_i$$

$$S = T e^{-i\frac{1}{\hbar} \int d\tau V_{\text{probe}} (\tau)}$$

- Gauss: $m \leq 2$

- $\langle e^{sX} \rangle = e^{\sum m \langle x^m \rangle}$

Field degrees of freedom

Effective two-atom description

Two atom Master equation

\[ \int \langle \hat{E}_i(t_1) \hat{E}_j(t_2) \rangle \]

trace out field degrees of freedom

\[ V_{\text{dip-dip}} \propto \sum_{i \neq j} \frac{e^{i \theta_{ij}}}{x_{ij}^3} \left[ (1 - \cos^2 \theta_{ij}) x_{ij}^2 + (1 - 3 \cos^2 \theta_{ij}) (ix_{ij} - 1) \right] \]
Can one expect superradiance?

The important parameter is

\[ n \lambda^2 r \]

optical depth

\( n \): density, \( \lambda \): wavelength, \( r \): system size
Can one expect superradiance?

The important parameter is

\[ n \lambda^2 r \]

or \[ n \lambda^3 \]?

\( n \): density, \( \lambda \): wavelength, \( r \): system size

\[ \dot{\rho}_{i,j} = \frac{i}{\hbar} \sum_{\mu} \rho_{\mu} \sum_{k=i,j} \left[ \sigma_{k\mu} \mathcal{E}_{L,\mu}^-(\vec{r}_k) + \sigma_{k\mu}^+ \mathcal{E}_{L\mu}^+(\vec{r}_k), \rho \right] \]

\[ + \frac{i}{\hbar} \sum_{\mu,\nu} \sum_{k=i,j} H_{k\mu,k\nu} \left[ [\sigma_{k\mu}, \sigma_{k\nu}^+] , \rho \right] \]

\[ - \sum_{\mu,\nu} \sum_{k,l=i,j} \frac{\Gamma_{k\mu,l\nu}}{2} \left( [\rho \sigma_{k\mu}, \sigma_{l\nu}^+] + [\sigma_{k\mu}, \sigma_{l\nu}^+ \rho] \right) \]

\[ - \sum_{\mu,\nu} \sum_{k,l=i,j} \frac{\Gamma_{k\mu,l\nu} + \gamma_{k\mu,l\nu}}{2} \left( [\rho \sigma_{l\nu}^+, \sigma_{k\mu}] + [\sigma_{l\nu}^+, \sigma_{k\mu} \rho] \right) \]
Master Equation

\[ \dot{\rho}_{i,j} = \frac{i}{\hbar} \sum_{\mu} \delta_{\rho \mu} \sum_{k=i,j}^{} \left[ \sigma_{k\mu} \mathcal{E}_{L,\mu}^-(\vec{r}_k) + \sigma_{k\mu}^+ \mathcal{E}_{L,\mu}^+(\vec{r}_k), \rho \right] \]

\[ + \frac{i}{\hbar} \sum_{\mu, \nu} \sum_{k=i,j}^{} H_{k\mu,k\nu} \left[ [\sigma_{k\mu}, \sigma_{k\nu}] \right] \]

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\[ - \sum_{\mu, \nu} \sum_{k,l=i,j}^{} \frac{\Gamma_{k\mu,l\nu} + \gamma_{k\mu,l\nu}}{2} \left( [\rho \sigma_{l\nu}^+, \sigma_{k\mu}] + [\sigma_{l\nu}^+, \sigma_{k\mu} \rho] \right) \]
\begin{equation}
\dot{\rho}_{i,j} = \frac{i}{\hbar} \sum_{\mu} \rho_{\mu} \sum_{k=i,j} \left[ \sigma_{k\mu} \mathcal{E}^{-}_{L,\mu}(\hat{r}_k) + \sigma_{k\mu}^{\dagger} \mathcal{E}^{+}_{L,\mu}(\hat{r}_k), \rho \right] \\
+ \frac{i}{\hbar} \sum_{\mu,\nu} \sum_{k=i,j} H_{k\mu,k\nu} \left[ [\sigma_{k\mu}, \sigma_{k\nu}^{\dagger}] \right] \rangle \langle \hat{E}(t) \rangle \\
- \sum_{\mu,\nu} \sum_{k,l=i,j} \frac{\Gamma_{k\mu,l\nu}}{2} \left( [\rho \sigma_{k\mu}, \sigma_{l\nu}^{\dagger}] \right) \\
- \sum_{\mu,\nu} \sum_{k,l=i,j} \frac{\Gamma_{k\mu,l\nu} + \gamma_{k\mu,l\nu}}{2} \left( [\rho \sigma_{l\nu}^{\dagger}, \sigma_{k\mu}] + [\sigma_{l\nu}^{\dagger}, \sigma_{k\mu} \rho] \right)
\end{equation}
New experimental systems: example

- Ultracold Rydberg atoms

(Phil Gould, Ed Eyler, Uconn)
Effective decay times from 40P into nS

\[ \tau_{\text{eff}} / \mu s \text{ (inverse Einstein A)} \]

In vacuum: decay into low n is favored
In dense gas: decay into high n is favored \( \Rightarrow \lambda \text{ large, } n \lambda^2 \text{ r large!} \)

superradiant decay!

(Daniel Vrinceanu)
Effective decay times from 40P into nS

- In vacuum: decay into low n is favored
- In dense gas: decay into high n is favored $\Rightarrow \lambda$ large, $n \lambda^2 r$ large!

superradiant decay!

Experimental Proof!

Superradiance in Rydberg systems

![Graph showing the number of Rydberg atoms over time. The graph compares theory and experiment. The start of measurement is marked with a dashed line.]
These lectures

• Cooperative effects in complex systems
  ‣ Collective (Lamb) level shifts
  ‣ Subradiance
  ‣ Entanglement

• New application: atomically thin mirrors
Decay dynamics

deexcited level population

fast     

slow

cf. two atoms

destructive interference

\[ \frac{1}{\sqrt{2}} (|eg\rangle - |ge\rangle) \equiv |0, 0\rangle \]
Subradiance?
Subradiant states

\[ J = \frac{N}{2} \quad J = \frac{N}{2} - 1 \quad J = 0 \]

Spontaneous superradiant decay
dipole-dipole interaction
Transitions

C = 31 R = 31: System Variables vs. Density Matrix Variables: line at max value of $\Gamma$ ($a=0.79398$)

excited state population

two-atom coherence
Subradiance: Outlook

- Dynamics of subradiance = transition to many-body localized state?
  ➡ Create, manipulate localization

- Engineered subradiance to create stable states without spontaneous decay
  ➡ Create, stabilize many-body entangled state (Dissipative non-equilibrium physics)
Collective Lamb shift

- "Lamb shift" is the result of interaction with the vacuum fluctuations.
- In the case of altered density of states of the "vacuum" (i.e., the surrounding space), the value of the shift changes.
- With a high (superradiant) density of radiators, the density of states inside the medium can be considerably altered.
Collective Shift

has spontaneous part....

\[ \gamma_{ij}(\omega) = \frac{\phi^2}{\hbar^2} \int d\tau \left\langle \left[ E_i^-(t), E_j^+(t + \tau) \right] \right\rangle e^{i\omega \tau} \]

\[ \Delta_{\text{spont}}^{(ij)} = \frac{1}{2\pi} \mathcal{P} \int d\omega' \frac{\gamma_{ij}(\omega')}{\omega - \omega'} \]

\[ \left\langle \left[ E^-, E^+ \right] \right\rangle \propto \left\langle \left[ a, a^\dagger \right] \right\rangle = 1 \]

independent on number of photons
Collective Shift

has spontaneous part....

\[
\gamma_{ij}(\omega) = \frac{g^2}{\hbar^2} \int d\tau \langle [E_i^-(t), E_j^+(t + \tau)] \rangle e^{i\omega\tau}
\]

\[
\Delta_{\text{spont}}^{(ij)} = \frac{1}{2\pi} \mathcal{P} \int d\omega' \frac{\gamma_{ij}(\omega')}{\omega - \omega'}
\]

\[
\langle [E^-, E^+] \rangle \propto \langle [a, a^\dagger] \rangle = 1
\]

independent on number of photons
Collective Lamb Shift

Collective Lamb Shift

cavity with variable thickness

Collective Lamb Shift

Collective Lamb Shift

Optical depth = highest

Optical depth = lowest
Collective Shift

The operator has a spontaneous part...

\[ \langle [E_i^-(t), E_j^+(t + \tau)] \rangle e^{i\omega \tau} \]

\[ \langle E^- E^+ \rangle \propto \langle a^+ a \rangle = n \]

dependent on number of photons

... and “stimulated” part

\[ \Gamma_{ij}(\omega) = \frac{\phi^2}{\hbar^2} \int d\tau \langle [E_i^-(t)E_j^+(t + \tau)] \rangle e^{i\omega \tau} \]

\[ \Delta_{\text{stim}}^{(ij)} = \frac{1}{2\pi} \mathcal{P} \int d\omega' \frac{\Gamma_{ij}(\omega')}{\omega - \omega'} \]
Collective Shift

has spontaneous part....

\[ \langle E^- E^+ \rangle \propto \langle a^\dagger a \rangle = n \]

dependent on number of photons

... and “stimulated” part

\[ \Gamma_{ij}(\omega) = \frac{\phi^2}{\hbar^2} \int d\tau \left\langle \left[ E_i^-(t), E_j^+(t + \tau) \right] \right\rangle e^{i\omega \tau} \]

\[ \Delta_{\text{stim}}^{(ij)} = \frac{1}{2\pi} \mathcal{P} \int d\omega' \frac{\Gamma_{ij}(\omega')}{\omega - \omega'} \]
Collective Shift: decay of inverted TLS

Plot of Superradiant Evolution for $C = 31$ Rho = 10
These lectures

• Cooperative effects in complex systems
  ▸ Collective (Lamb) level shifts
  ▸ Subradiance
  ▸ Entanglement

• New application: atomically thin mirrors
Superradiance and Entanglement

Does (Dicke) superradiance need/create entanglement?

NO
Example: 2-atom Dicke

- PPT (Peres-Horodecki) criterion:
  
  Eigenvalues of partial positive transpose $\geq 0$

$$\rho = \sum_{ijkl} p_{kl} |i\rangle\langle j| \otimes |k\rangle\langle l|$$

$$\rho_{\text{PPT}} = \sum_{ijkl} p_{kl} |i\rangle\langle j| \otimes |l\rangle\langle k|$$
Superradiance and Entanglement

How to define/calculate many-particle entanglement?

O Gunne, G Toth
Multipartite entanglement in four-qubit cluster-class states

YK Bai, ZD Wang
Exact and asymptotic measures of pure-state multipartite entanglement

CH Bennett, S Popescu, S Pironio, V Scarani, JA Smolin
Geometric measures of entanglement and applications to bipartite and multipartite quantum states

TC Wei, PM Goldbart
Scalable multiparticle entanglement of trapped ions

H Häffner, W Hänsel, CF Roos, J Benhelm, M Chwalla

~ 10,000 for “definition of multipartite entanglement”
Superradiance and Entanglement

Does (Dicke) superradiance need/create entanglement? (Initial state: no entanglement)

Dicke superradiant time evolution = separable states

constructive proof

Wolfe, Yelin, PRL 112, 140402 (’14)
Superradiance and Entanglement

Does (Dicke) superradiance need/create entanglement? (Initial state: no entanglement)

\[
\text{Dicke superradiant states} = \text{separable states}
\]

Wolfe, Yelin, Wolfe, Yelin, PRL 112, 140402 ('14)
Superradiance and Entanglement

Does (Dicke) superradiance need/create entanglement? (Initial state: no entanglement)

Our system: mixed state of $N$-atom Dicke states with $N+1$ known independent coefficients $p_i$

Compare to mixture of symmetric product states of $N$ (two-level) atoms (needs $N+1$ coefficients $y_i$)

$(N+1)$ - dim. equation system

Wolfe, Yelin, Wolfe, Yelin, PRL 112, 140402 ('14)
Form of equations

• General Dicke states:

\[ \rho_{\text{GDS}} = \sum_n \chi_n |D_n \rangle \langle D_n| \]

• Separable diagonally symmetric:

\[ \rho_{\text{SDS}} = N! \sum_n \sum_{j=1}^{j_{\text{max}}} \frac{x_j y_j^{n_0} (1 - y_j)^{n_1}}{n_0! n_1!} |D_n \rangle \langle D_n| \]
Superradiance and Entanglement

Does (Dicke) superradiance need/create entanglement? (Initial state: no entanglement)

our system: mixture of N-atom Dicke states with N+1 known independent coefficients

compare to mixture of symmetric product states of N (two-level) atoms (needs N+1 coefficients $y_i$)

condition: all coefficients $0 \leq p_i \leq 1$

Wolfe, Yelin, PRL 112, 140402 (’14)
Superradiance and Entanglement

Does (Dicke) superradiance need/create entanglement? (Initial state: no entanglement)

our system: mixed state of N-atom Dicke states with N+1 known independent coefficients \( p_i \)

\[ \text{condition: all coefficients } 0 \leq p_i \leq 1 \]

\( (N+1) \)-dim. equation system

\[ \text{comparing mixture of symmetric product states of N (two-level) atoms (needs N+1 independent coefficients } y_i \)} \]

(works!)
Superradiance and Entanglement

Driven superradiant system:

• Driving alone does not entangle atoms
• Superradiance alone does not entangle atoms
• Driving and superradiance together entangle atoms!
Fuzzy Bunny?
Spin Squeezing

• Correlated ("squeezed") spins could improve resolution in one direction ("quadrature").
(Spin) Squeezing

• How to measure squeezing/measurement improvement?

\[ \xi^2 \equiv \frac{\text{optimal variance}}{\text{unsqueezed optimal variance}} \]
Spin squeezing

Old problem: How to improve metrology by spin squeezing ensembles

Groups of Bigelow, Kuzmich, Lewenstein, Mølmer, Polzik, Sanders, Sørensen, Vuletic, Wineland,...
Spin Squeezing

• Correlated ("squeezed") spins could improve resolution in one direction ("quadrature").

Kitagawa, Ueda, PRA 47, 5138 (93)
• How to measure squeezing/measurement improvement?

\[
\xi^2 \equiv \frac{\text{optimal variance}}{\text{unsqueezed optimal variance}}
\]

\[
\xi^2 = \frac{N}{2} \left[ \langle J_1^2 + J_2^2 \rangle - \sqrt{\langle J_1^2 - J_2^2 \rangle^2 + \langle J_1 J_2 + J_2 J_1 \rangle^2} \right]
\]

(J_1, J_2, are uncertainties in the two directions orthogonal to the total spin J)
Superradiant Spin Squeezing

\[ N = \ \text{5} \ \text{10} \ \text{15} \ \text{20} \]

\[ \xi^2 \]

\[ \Omega \ (\text{driving field}) \]

Superradiant Spin Squeezing

\[ \xi^2 \text{ vs. } \Omega/N \]

\( N = 2, 4, 8, 16, 32, 64 \)

\[ \xi^2 \]

Best case for Dicke ensemble

What about realistic systems?

- Dicke: 3 parameters \((N, \Gamma, \Omega)\)
- Realistic systems: \((OD, \text{rel. density}, \Gamma, \Omega, \gamma_{ij}, \Delta, \delta_{ij})\)
- Is it possible to find parallels?
- minimize “incoherent” aspects?

➡ key: spontaneous decay, shift instead of induced!
Spin squeezing in realistic systems?
Spin squeezing in realistic systems?
Thank you!